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Discussion

Discussions on "On the number of modes required for statistical energy analysis-based calculations"

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1. Introduction

In Ref. [1], it was argued using two subsystems that "for the applicability of statistical energy analysis (SEA), what is important is the presence of large number of modal pairs in the interacting subsystems and not large number of modes in each subsystem". It was further claimed that "Even if there is only one mode present in a particular subsystem, if the interacting subsystem has several modes in the frequency band, SEA is applicable". The purpose of this discussion is to highlight that the conclusions made in Ref. [1] cannot be generalized to using SEA for the analysis of a system with more than two subsystems and such generalization might incur substantial errors.

2. Key assumptions of SEA

SEA was originally developed based on the power flow analysis of two coupled oscillators. The well-known expression regarding the time-averaged power flow between the two oscillators is

$$\bar{P}_{12} = g_{12}(\bar{e}_1 - \bar{e}_2), \tag{1}$$

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where $\bar{e}_1$ and $\bar{e}_2$ are the time-averaged total vibrational energies of the two oscillators, respectively, and g_{12} is the constant of proportionality as given in Ref. [2]. When extending Eq. (1) to coupled groups of oscillators, namely subsystems in SEA, normally three assumptions were made [2–4]:

- The time-averaged total vibrational energy in a subsystem is equally distributed over all of the modes in the frequency band ("equipartition of energy"), and the modal responses are incoherent so that energy sums apply;
- Since at resonance, the magnitude of constant of proportionality g_{12} for each modal pair is primarily controlled by the damping of the oscillators, damping values are assumed to be similar for all the modes within a subsystem and frequency band so that g_{12} is similar for all the modal pairs; and
- The number of modes is sufficient and equally distributed in the frequency band.

Consider two coupled subsystems with N_1 modes in subsystem 1 with N_2 modes in subsystem 2 and assume that resonant modes can be described by simple oscillators. Then by applying the first two assumptions above, a relationship similar to Eq. (1), relating the total power flow between the two subsystems, $\bar{\Pi}_{12}$, and the averaged modal energies in each subsystem, $\langle \bar{e}_1 \rangle_{N_1}$ and $\langle \bar{e}_2 \rangle_{N_2}$, can be obtained [2],

$$\bar{\Pi}_{12} = \langle g_{12} \rangle_{N_1 N_2} N_1 N_2 \cdot \left[\langle \bar{e}_1 \rangle_{N_1} - \langle \bar{e}_2 \rangle_{N_2} \right], \tag{2}$$

where $\langle \rangle_{N_1 N_2}$ represents the average over all the modal pairs. By defining the coupling loss factor as $\eta_{12} = \langle g_{12} \rangle_{N_1 N_2} N_2/\omega$, Eq. (2) becomes,

$$\bar{\Pi}_{12} = \omega \eta_{12} N_1 \left[\frac{\bar{E}_1}{N_1} - \frac{\bar{E}_2}{N_2} \right], \tag{3}$$

where $\bar{E}_1$ and $\bar{E}_2$ are the total energies of the two subsystems, respectively. As the third assumption was not employed, it would appear that the requirement of sufficient number of modes in each subsystem was irrelevant for Eq. (3) to be valid. This is, however, only true for a system with only two coupled subsystems. Previous studies have already shown that Eq. (3) is generally valid for a two-subsystem assembly even though the coupling is strong for which few modes are in the subsystems [5]. However, for a system with more than two coupled subsystem, the third assumption is necessary for Eq. (3) to be valid generally within the system.

3. Three coupled oscillators

To help clarify the importance of the third assumption, consider three serially coupled oscillators as shown in Fig. 1. A detailed analysis of the power flow between the oscillators made by Sun et al. [6] showed that the time-averaged net power flow from oscillators 1 to 2 is given by,

$$\bar{P}_{12} = \beta_{12}(\bar{e}_1 - \bar{e}_2) + \gamma_{13}(\bar{e}_1 - \bar{e}_3), \tag{4}$$

where $\bar{e}_i$ is the time-averaged energy of oscillator i ($i = 1, 2, 3$), while β_{12} and γ_{13} are constants of proportionality, both depending on the parameters (mass, damping and stiffness) of all oscillators. This equation can be easily justified by assuming a very large m_3 with asymptotic

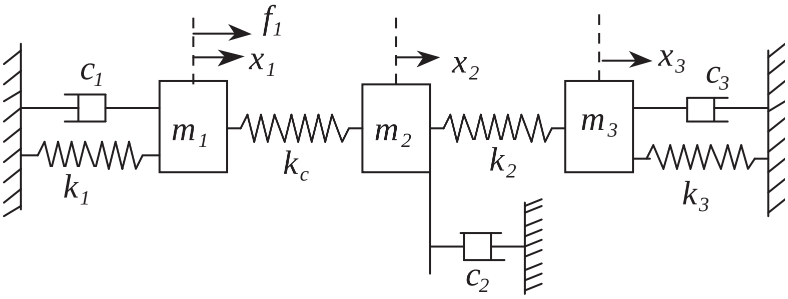

Fig. 1. Three oscillators coupled serially by spring elements.

solutions of β_{12} and γ_{13} expected to be g_{12} and 0, respectively. Eq. (4) indicates that the power flow between three series coupled oscillators generally consists of two parts, the direct power flow $\bar{P}_d = \beta_{12}(\bar{e}_1 - \bar{e}_2)$ between the oscillators which are physically coupled; and the indirect power flow $\bar{P}_i = \gamma_{13}(\bar{e}_1 - \bar{e}_3)$ between the oscillators which are not physically coupled. Although it was shown that reciprocity still holds for either direct or indirect power flow, i.e. $\beta_{12} = \beta_{21}$, $\gamma_{13} = \gamma_{31}$, the constant of proportionality β_{12} appears to be different from g_{12} simply because β_{12} is a function not only of the directly coupled oscillators and coupling parameters, but also that of the indirectly coupled oscillators and coupling parameters.

The dependence of the constant of proportionality β_{12} on all oscillator parameters adds difficulties in identifying the key parameters affecting the modal behaviour and characterizing the power flow in the system explicitly. However, numerical studies showed that the direct power flow between oscillators 1 and 2 is maximum when a resonance occurs between the two oscillators. More importantly, when a resonance occurs between oscillators 1 and 2, the ratio of indirect power flow to the direct power flow reaches a minimum, the indirect power flow between oscillators 1 and 3 being orders of magnitude less than the direct power flow between oscillators 1 and 2, as shown in Ref. [6]. A more general conclusion that could be reached here was that, between the two directly coupled oscillators, the direct power flow is associated with resonant transmissions; while the indirect power flow is with non-resonant transmissions. The result is significant as it shows that the indirect power flow may not be important when a resonance occurs between directly coupled oscillators. In this case, the three series coupled oscillator could be simplified to the two coupled oscillators model, or in other words, effects of the oscillators other than the directly coupled two on the direct power flow could be ignored.

Moreover, it is interesting to note that decreasing k_2 has similar effects as increasing m_2 on the constants of proportionality β_{12} and γ_{13}. Therefore, when k_2 is small, β_{12} and γ_{13} should be approaching g_{12} and 0, respectively, indicating that the direct power flow may not be affected if the indirect coupling strength is weak [6].

Given as a conceptual illustration, Fig. 2 shows three typical cases associated with the coupling of oscillators in energy sharing. If the natural frequencies of oscillators 1 and 2 are within the frequency band Δf, then resonant transmission occurs between oscillators 1 and 2, and the corresponding power flow is so dominant that the power flow between oscillators 1 and 3 could be neglected. If the resonance between oscillators 1 and 2 does not occur within the frequency band, the power flow between indirectly coupled oscillators 1 and 3 might not be ignored. In this case,

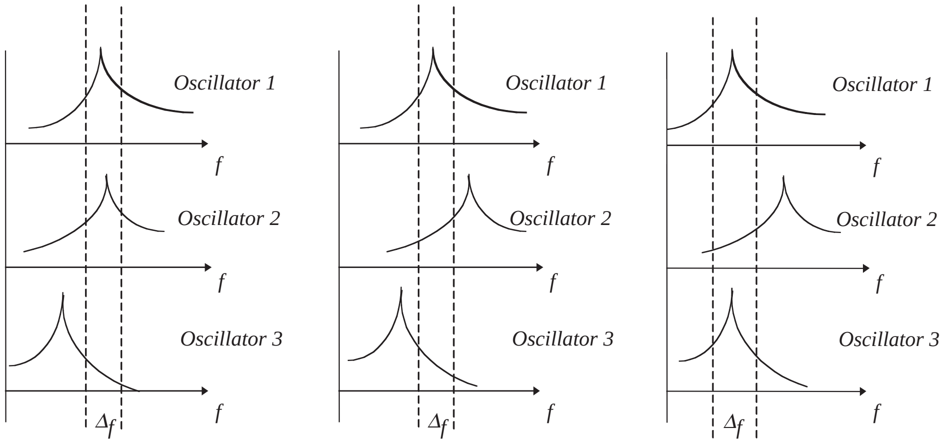

Fig. 2. Coupling between the oscillators.

depending on whether a resonance occurs between the oscillators 1 and 3 (oscillator 2 acts as a coupling element), the indirect power flow may also be characterized as resonant or non-resonant transmissions accordingly. However, unlike the direct power flow which is always dominated by the resonant transmission between directly coupled oscillators (especially for small damping values), the non-resonant transmission between indirectly coupled oscillators could play an important role in the indirect power flow.

For a system with multi-groups of oscillators (i.e. each group of oscillators representing the resonant modes of each subsystem), Eq. (3) cannot be extended directly because the indirect power flow is associated with the non-resonant transmission through the intermediate oscillator, and the energy sum might not apply. However, by assuming that each group of oscillators satisfies the three key assumptions made above, and, resonance spreads over the frequency band Δf between any pair of the modes in directly coupled groups of oscillators to ensure that the indirect power flows within the system are negligible, then Eq. (3) can be applied to any directly coupled oscillator groups. This result thus forms the basis of the classical SEA theory in which only resonant transmissions between directly coupled oscillators are considered. The third assumption is necessary to ensure resonance occurs within the frequency band. Quantitatively, the resonance condition requires that there are at least $N = \Delta f/(f\eta)$ modes equally distributed within the frequency band Δf, where f and η, respectively, are the averaged natural frequency and damping loss factor of the modes within this band. This is equivalent to requiring that the modal overlap $M(= nf\eta)$ to be greater than unity, where n is the modal density of a subsystem, that is, sufficient number of modes is required in each subsystem.

For a system with more than two subsystems, if there are not sufficient modes in the subsystem and the resonance condition is not fully complied, indirect power flows may exist [3,4,7]. For indirect resonance transmissions, by generally assuming that the modal responses are statistically independent, an expression similar to Eq. (3) for the indirect power flow might be obtained as

$$\bar{\Pi}_{13} = \omega\eta_{13}N_1\left[\frac{\bar{E}_1}{N_1} - \frac{\bar{E}_3}{N_3}\right], \tag{5}$$

where subscript '13' denotes an indirect transmission path, and $\eta_{13} = \langle \gamma_{13} \rangle_{N_1 N_3} N_3/\omega$, in which N_3 is the number of oscillators (i.e. resonant modes) in the subsystem indirectly coupled to group 1. In this case, we might have

$$N_1 \eta_{13} = N_3 \eta_{31}. \tag{6}$$

A typical example of energy transfer due to indirect resonant transmissions is a limp panel (no stiffness, thus no resonance) installed between two large acoustic cavities with rigid walls, the two cavities being the resonant subsystems.

For the indirect non-resonant transmission, however, Eqs. (5) and (6) might not hold because the modal responses are likely coherent, for which the energy sums may not apply. For practical structures, this normally happens at low frequencies where only few modes are within the frequency band of interest.

4. Conclusions

Consistent with the discussions above, Mace [7] characterized three types of analyses, the classical SEA, the quasi-SEA or SEA like analysis, and the energy flow analysis, in terms of the behaviour of the loss factor matrix. Specifically, the classical SEA only considers resonant transmissions between directly coupled subsystems. No indirect coupling paths exist in the model. The quasi-SEA or SEA-like analysis includes direct coupling paths and indirect coupling paths as well. The reciprocity relationship is still valid for all power flow paths, indicating that only resonant transmissions, either direct or indirect, are considered. The energy flow analysis basically includes all the power transmission paths, not only the resonant but also the non-resonant transmissions, in the model. Thus the loss factor matrix is generally full and not necessarily symmetrical. Mace [8] also pointed out that Eq. (3) is exact for a system comprising two subsystems. For a system with arbitrary number of subsystems, unless the 'weak coupling' condition is satisfied, an unique power flow relationship like Eq. (3) might not hold within the system such that the dynamics of the system has to be solved as a whole, which is therefore contrary to the spirit of SEA [8]. Langley [9] gave a detailed overview of the assumptions that are fundamental to the validity of SEA and the conditions that help to promote the behaviours in compliance with the assumptions. Basically, sufficient number of modes in the subsystems and equipartition of energy among those resonant modes are the two assumptions fundamental to a valid classical SEA analysis.

References

[1] K. Renji, On the number of modes required for statistical energy analysis-based calculations, *Journal of Sound and Vibration* 269 (2004) 1128–1132.

[2] C.B. Burroughs, R.W. Fischer, F.K. Kern, An introduction to statistical energy analysis, *Journal of the Acoustical Society of America* 101 (4) (1997) 1779–1789.

[3] D.E. Newland, Power flow between a class of coupled oscillators, *Journal of the Acoustical Society of America* 43 (3) (1968) 553–559.

[4] R.S. Langley, A derivation of the coupling loss factors used in statistical energy analysis, *Journal of Sound and Vibration* 141 (2) (1990) 207–219.
[5] E.K. Dimitriadis, A.D. Pierce, Analytical solution for the power flow exchange between strongly coupled plates under random excitation: a test of statistical energy analysis concepts, *Journal of Sound and Vibration* 123 (3) (1988) 397–412.
[6] J.C. Sun, C. Wang, Z.H. Sun, Power flow between three series coupled oscillators, *Journal of Sound and Vibration* 189 (2) (1996) 215–229.
[7] B.R. Mace, Statistical energy analysis, energy distribution modes and system modes, *Journal of Sound and Vibration* 264 (2003) 391–409.
[8] B.R. Mace, On the statistical energy analysis hypothesis of coupling power proportionality and some implications of its failure, *Journal of Sound and Vibration* 178 (1994) 95–112.
[9] R.S. Langley, SEA: current and future research needs, *Proceedings of the First International AutoSEA Users Conference*, San Diego, CA, USA, July 27–28, 2000, pp. 126–135.